NAME_____

DATE_____

Part 0 - Drawing an ellipse.

You will need: 1 piece of graph paper
1 ruler
1 piece of string between 10 and 12 inches long
2 push pins
scissors

1.   Draw a line across the paper lengthwise through its center
(since the paper is 11" long, measure 5 1/2" to find this line)
Draw a line across the paper widthwise through it center
(since the paper is 8 1/2" wide, measure 4 1/4" to find this line)
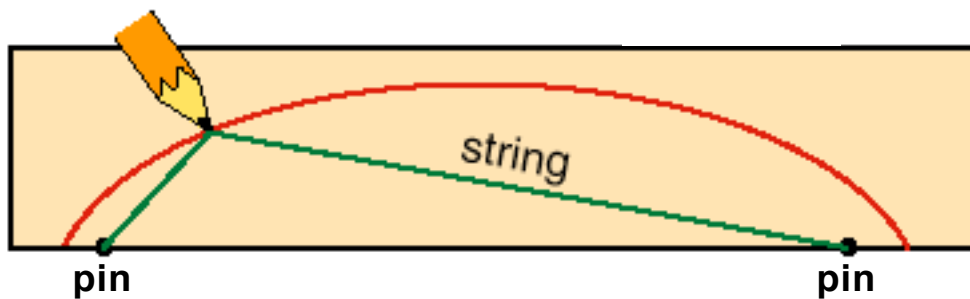The two lines should cross roughly at the center point of the paper

2.   Holding the paper like this,
count off 10 blocks on either
side of the center point. These
will be your **focus** points.

3.   Stick a push pin through each focus point. Make sure they are firmly in place.
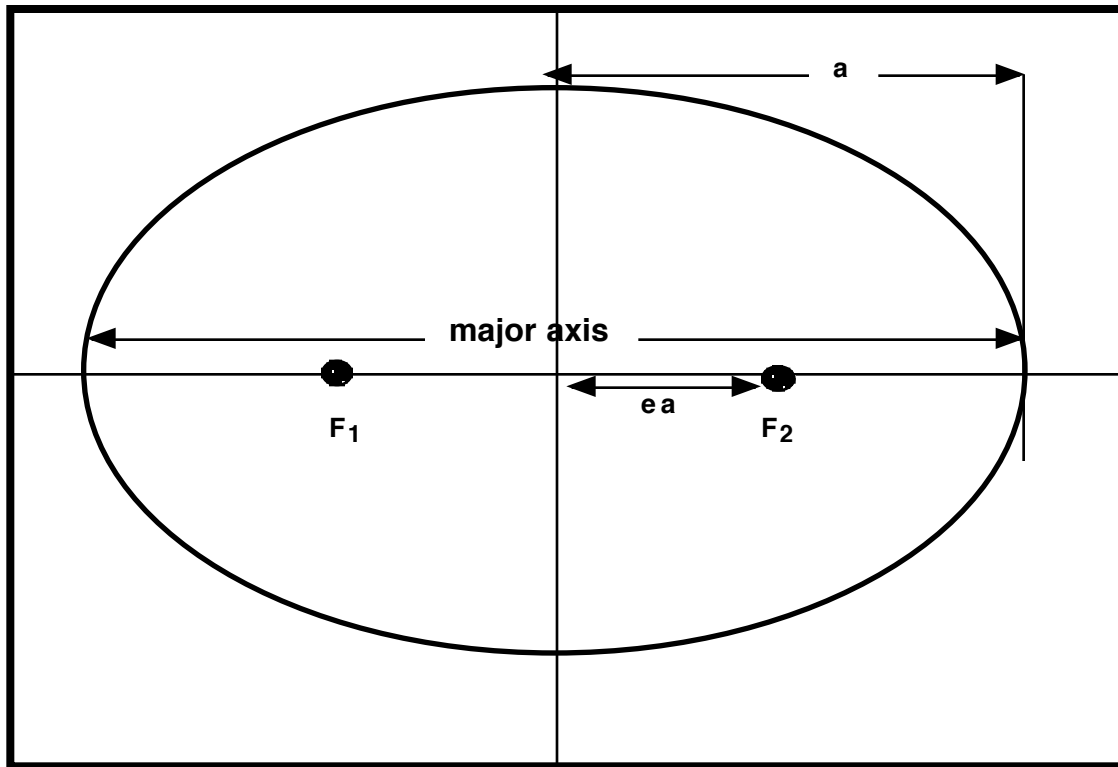
4.   Take the string and tie it into a loop with as little excess as possible.

5.   Place the loop of string flat on the paper around the outside of <u>both</u> pins.

6.   Using a pencil, place the tip of the pencil inside the loop, and place it so that it stretches the loop tightly, with the other end being pulled by a push pin. Then, carefully trace the   ellipse on your graph paper by keeping maximum tension in the string between the tip of the pencil and one (or both) pins. (Make sure that the string is looped around the <u>bottom</u> of the pin, not the upper plastic part.)   The pencil should trace out a smooth, curved path that eventually forms a closed oval-shaped figure called an **ellipse**.

pin   **pin**

7. Label the following on your ellipse:

8. The two focus points lie on a line that passes through the widest part of the ellipse. This line is called the **major axis**. (The central line, perpendicular to the major axis, cutting the ellipse in half, is called the **minor axis**.)
Half of the major axis (called the *semi-major axis*) is labelled "**a**". Therefore, the major axis has the length "**2a**".

ECCENTRICITY

Eccentricity is a measure of how "elongated", or stretched out, the ellipse is. The word "eccentric" literally means "off-center", and is sometimes used to describe unusual human behavior.
In the case of an ellipse, eccentricity ("**e**") is a mathematical factor which represents the fraction of "a" that either focus is from the center. Therefore, the distance from the center point to either focus point can be called "**ea**".
To find e, simply **divide ea by a.**
"e" should be a number between 0 and 1   (0 < e < 1).
Measure and calculate the eccentricity (e) of your ellipse.

What would happen to the shape of the ellipse if the two focus points (called foci) are **brought <u>closer together</u>** (in other words, letting e approach 0)?

What would happen to the shape of the ellipse if the foci are moved **<u>farther apart</u>** (in other words, letting e approach 1)?

WHAT MAKES AN ELLIPSE DIFFERENT FROM AN OVAL?

An ellipse is a member of a family of curves known as ***conic sections*** (derived from a right-angle cone). It has certian mathematical properties that distinguish it from all other curves that might look similar. Here is how you can distinguish an ellipse from a similar-looking oval.

Using the ellipse you made, draw a line from **F₁** to *any point on the ellipse* (call it "**p**"). This line is called a "**radius vector**". Label it "**r₁**".

Now, draw another radius vector from **p** to **F₂**. Label it "**r₂**".

Measure the length of **r₁** and **r₂**. The sum of these lengths will always equal **2a** (the length of the major axis)! IN EQUATION FORM:

$$\boxed{r_1 + r_2 = 2a}$$ **(and, of course, 2a is a constant)**

Try this again, using a different point on the ellipse. You should get the same sum (2a).

KEPLER'S LAWS OF PLANETARY MOTION

I. Law of Ellipses.

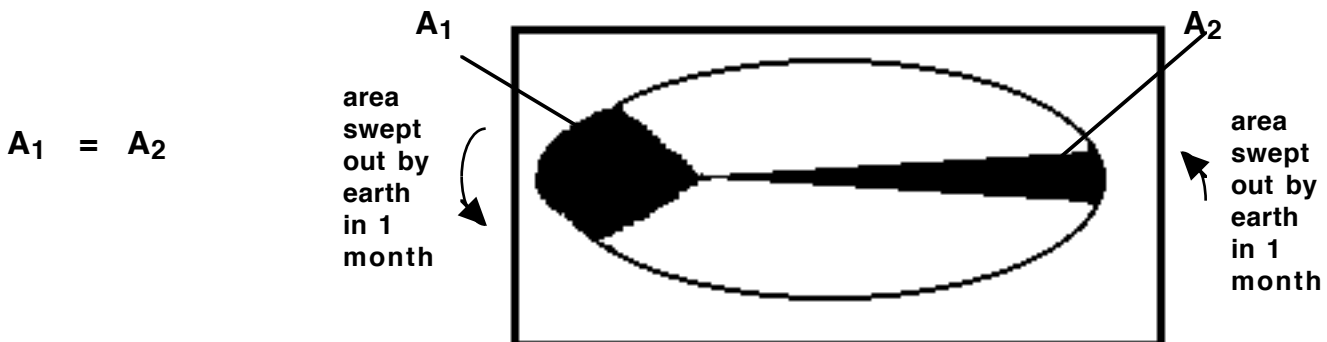*Planets orbit the sun in elliptical orbits, with the sun at one focus point.*

This law was derived after careful analysis of years of data taken by the Danish astronomer, Tycho Brahe about the positions of the planet Mars. Unfortunately, Brahe was not a Copernican, and thought that the Earth was in the center of the universe, orbited by the moon, with the sun going around the Earth, and the other planets going around the sun. It was known as the Tychonic System. Not being the mathematician that Johannes Kepler was, and, of course, since this system was wrong, he was never able to establish proof of his theory from his data. On his death bed, he pleaded for Kepler to use the data (the best ever taken up to that point in time) to prove his theory. Of course, Kepler had his own agenda and used the data to eventually establish the most precise set of rules governing the motions of the planets ever assembled.
It was especially scorned for being contrary to the belief that all objects in the heavens travel in circular paths - even going against the Copernican rule for planetary motion.

II. Law of Equal Areas.

Planets traveling around the sun sweep out equal areas during equal time intervals.

Essentially, this law says that planets travel faster when they are closer to the sun than when they are farther from the sun . Since Kepler established in his first law that the planets' orbits are not circular, but elliptical, with the sun at one focus, the distance between the planet and the sun changes constantly. When the planet is closest to the sun (at "perihelion"), it is "whipped around the sun", until it is at its farthest point from the sun (at "apehelion"), where it has slowed down, having just enough speed to make its way back toward the sun. For example, a radius vector drawn from the sun to the earth will "sweep out" the same area (resembling a pie wedge) in say, 30 days, that it will during any 30 day interval any time of the year around the sun.

A₁   **A₂**

area swept out by earth in 1 month

A₁ = A₂

area swept out by earth in 1 month

ECCENTRICITY GREATLY EXAGGERATED

III. Law of Equal Revolutions.

The cube of the average distance a planet is from the sun varies directly with the square of its period of revolution.

This law was developed by Kepler many years after the first two, and is obviously the most mathematical of the three. It essentially says that the farther away a planet is from the sun, the longer it will take to complete one revolution. In other words, planets that are closer to the sun travel faster than planets farther away. Recalling the math we did earlier in the year, this law specifically states that:

$$R^3 \quad \alpha \quad T^2$$

R -> average distance from a planet to the sun
T -> time it takes that planet to revolve once

therefore,

$$R^3 = kT^2$$

which means that

$$\frac{R^3}{T^2} = k$$

Since "k" is a constant, this means that if you divide the cube of the average distance of any planet from the sun, and divide this by the square of the time it takes to go around the sun, it will come out to the same number for *every planet*!!

This discovery not only represented a tremendous achievement for Kepler, but an insight into the motion of the planets so precise that Isaac Newton would use these laws to develop his **Theory of Universal Gravitation**, thus explaining *WHY* the planets moved the way they did. Kepler and Galileo, who lived at the same time (and actually corresponded with each other, although they never met), paved the way for the science of physics to be formalized one generation later by Newton. They could be considered to be **KINEMATICIANS** - analyzing the motions of objects on Earth and in space only.

Newton would go on to say, ". . . If I have seen far, it is because I have stood on the shoulders of giants."
He was obviously referring to the accomplishments of Galileo and Kepler, as well as Copernicus, Descartes, Brahe and all the others who made it possible for him to define the cause of all motion in the universe so strongly that it could not possibly be disputed at that time.

ECCENTRICITY OF ORBITS IN OUR SOLAR SYSTEM

Fortunately, Kepler chose the planet Mars in his analysis of planetary motion. Mars has the highest eccentricity of the 5 planets known at that time. Still, its "e" value is only 0.09! (Its relatively high eccentricity is probably due to the gravitational influence of Jupiter early in the formation of the solar system.) Jupiter, the most massive planet in the solar system, lies just beyond Mars after the asteroid belt. Earth's eccentricity is only 0.03. It is closest to the sun during our winter season, at a distance of 91,500,000 miles; it is farthest during our summer season, at a distance of 94,500,000 miles. (Remember that seasons are NOT determined by our distance to the sun, but because of the tilt of Earth's axis.) Had Kepler chosen Venus, Jupiter or Saturn, he might not have come up with a significant enough eccentricity to question the circular theory of orbits, and may not have developed his First Law.
However, we know today the the planet Pluto has a very highly eccentric orbit, which actually takes it closer to the sun than Neptune at perihelion. Comets also tend to have extremely eccentric orbits, many of which take them very close to the sun at perihelion, and well beyond Pluto at aphelion! But whether following a highly eccentric orbit or not, all objects in our solar system follow all three of Kepler's Laws.