WORCESTER POLYTECHNIC INSTITUTE
TWENTY-EIGHTH INVITATIONAL MATH MEET
OCTOBER 20, 2015
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes long. Questions 1-4 are each worth 1 point. Questions 5-8 are each worth 2 points. Questions 9-11 are each worth 3 points. Calculators and other electronics **MAY NOT** be used.

1 point each:

1. Find **k** so the following system is consistent:

   $x + y = 1$

   $kx + y = 2$

   $x + ky = 3$

2. Simplify $(63! - 61!) \bmod 71$

3. A square 1 cm on a side has the corners cut off resulting in a regular octagon. What is the length of a side of the octagon?

4. Convert $89\frac{1}{8}$ into binary form.

2 points each:

5. Find the prime factors of **1,000,027** .

6. Compute the value of the continued fraction.

$$\cfrac{1}{5+\cfrac{1}{5-\cfrac{1}{5+\cfrac{1}{5-\ldots}}}}$$

7. For the conic section $\frac{(y+3)^2}{9} - \frac{(x-7)^2}{25} = 1$ please describe its asymptotes.

8. Consider the graph of the discrete function $f(n) = \sum_{k=1}^{n} (3/7)^k$.

What will it have for a horizontal asymptote?

3 points:

9. Reduce the fraction $\frac{116{,}690{,}151}{427{,}863{,}887}$ to lowest terms.

10. Simplify $\frac{(4+\sqrt{15})^{3/2} + (4-\sqrt{15})^{3/2}}{(6+\sqrt{35})^{3/2} - (6-\sqrt{35})^{3/2}}$ to a rational fraction.

11. Find a number system so that the number **11111** in that system is a perfect square of another number in that system. Your answer should be the **base** for that number system.

NAME_____

SCHOOL_____

WORCESTER POLYTECHNIC INSTITUTE

TWENTY-EIGHTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 20, 2015
INDIVIDUAL EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	**k = 4**		**5**	**7*19*73*103**	
2	**0**		**6**	$L = \frac{27 - \sqrt{27^2 - 100}}{10}$	
3	$\sqrt{2} - 1$		**7**	**y = -3 +/- (3/5)(x-7)**	
4	**1011001.001_2**		**8**	y = ¾	
# CORRECT × 1 =			# CORRECT × 2 =		

QUESTION	ANSWER	SCORE
9	**3/11**	
10	**7/13**	
11	**base = 3** (102^2_3)	
# CORRECT × 3 =		

Individual Total