WORCESTER POLYTECHNIC INSTITUTE

31st INVITATIONAL MATH MEET
October 16, 2018
TEAM EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. All 14 problems are count equally. Calculators and other electronics **MAY NOT** be used.

Problem 1. 2.0161616... is rational. Suppose it equals $\frac{a}{b}$ in reduced form. What is $a + b$?

Ans: 2986

Problem 2. What is the largest positive integer to divide $n^5 - n$ for any integer n?

Ans: 30

Problem 3. Express the sum $\sin(100\pi t) + \sin(110\pi t)$ as the product of trig functions.

Ans: $2\sin(105\pi t)\cos(5\pi t)$

Problem 4. The infinite geometric sequence that begins $2^{300}, 2^{298}, 2^{296}, 2^{294}...$ contains only one odd integer. If this odd integer is the n-th term of the sequence, what is n?

Ans: $n = 151$

Problem 5. There are two real values of s for which $x^4 - x^3 - 18x^2 + 52x + k$ has factors of the form $(x - s)$ for some k. One is $s = 2$. What is the other?

Ans: $s = -5$

Problem 6. What is the exact numerical value of $\csc(20°) - \cot(40°)$?

Ans: $\sqrt{3}$

Problem 7. In a 12 by 5 rectangle, a diagonal is drawn and circles are inscribed in both of the resulting right triangles. What is the distance between the centers of these circles?

Ans: $\sqrt{65}$

Problem 8. Evaluate and simplify $\cos(36°)\cos(108°)$.

Ans: $-\frac{1}{4}$

Problem 9. What is the multiplicative inverse of $\mathbf{7^{1227}}$ **mod 13**?.
Note: Your answer must be an integer from 0 to 12.

Problem 10. What are all real values of p for which the inequalities

$$-3 < \frac{x^2 + px - 2}{x^2 - x + 1} < 2 \tag{1}$$

is satisfied by all real values of x?

Ans: $-1 < p < 2$

Problem 11. In a circle whose diameter is 16, two parallel chords partition the circle into 4 congruent arcs. What is the length of one of these chords?

Ans: $8\sqrt{2}$

Problem 12. In an arithmetic progression, the sum of the third and fifth terms is 14 and the sum of the first 12 terms is 129. If the n-th term is 193, what is the value of n?

Ans: 128

Problem 13. What are all real numbers x for which $\log_{2x+4}(x^2 + 1) \leq 1$?

Ans: $-2 < x < -\frac{3}{2}$ and $-1 \leqslant x \leqslant 3$

Problem 14. Consider the set of all real (x, y, z) such that $x^2 - 6x + y^2 + 10y + z^2 - 12z = 99$. What point in this set has the largest z value?

Ans: **3,-5,19**